Homework 7 - Solutions

Math 2L03

1. (a) False. Remember the example $f(x) = x^3$ and $x = 0$.

   (b) False.

   (c) False. Note that $f(x) = \sin x$ is an odd function, but $f'(x) = \cos x$ is an even function.

   (d) True.

   Recall that a function is even if for all x in the domain of f,

   $$f(-x) = f(x)$$

   Now taking derivatives of both sides in the above equation we get that

   $$-f'(-x) = f'(x)$$

   which means that the derivative is an odd function. Note that we had to use chain rule when computing the derivative of the left hand side of the equality.

2. (a) $x = 4$.

   (b) $x = n\pi$, where n is an integer.

   (c) $x = 0, 2$.

3. (a) Absolute maximum: $f(0) = 9$, Absolute minimum: $f(-3) = -72$

   (b) Absolute maximum: $f(3) = 1/2$, Absolute minimum: $f(-1/3) = -9/2$

4.

5.

6. Length and Width are 20cm.

7. $x = 100$

8. $1440\pi \ cm^3$

9. (a) Marginal Cost: $C'(x) = 100 - 0.06x + 0.000012x^2$

Revenue: $R(x) = 200x - 0.05x^2$

Marginal Revenue: $R'(x) = 200 - 0.1x$

Marginal Profit : $P'(x) = 100 - 0.04x - 0.000012x^2$

$C'(250) = 85.75$ (dollars per unit)

$C'(750) = 61.75$ (dollars per unit)

$R'(250) = 175$ (dollars per unit)

$R'(750) = 125$ (dollars per unit)

$P'(250) = 89.25$ (dollars per unit)

$P'(750) = 63.25$ (dollars per unit)

(b) The marginal cost/revenue/profit at 250 is approximately the cost/revenue/profit of producing the 251st item.

(c) $x = 1667$.